Extraction of Δg from spin asymmetries in pp scattering

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RBRC & BNL Nuclear Theory

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Outline:

- Introduction
- Next-to-leading order predictions for spin asymmetries at RHIC
- Technique for “global analysis”
- Conclusions

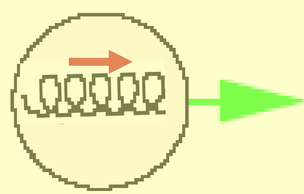
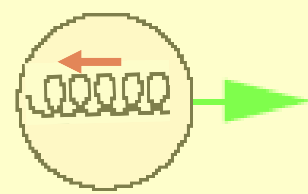
I. Introduction

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

Quark and gluon orbital ang. mom.

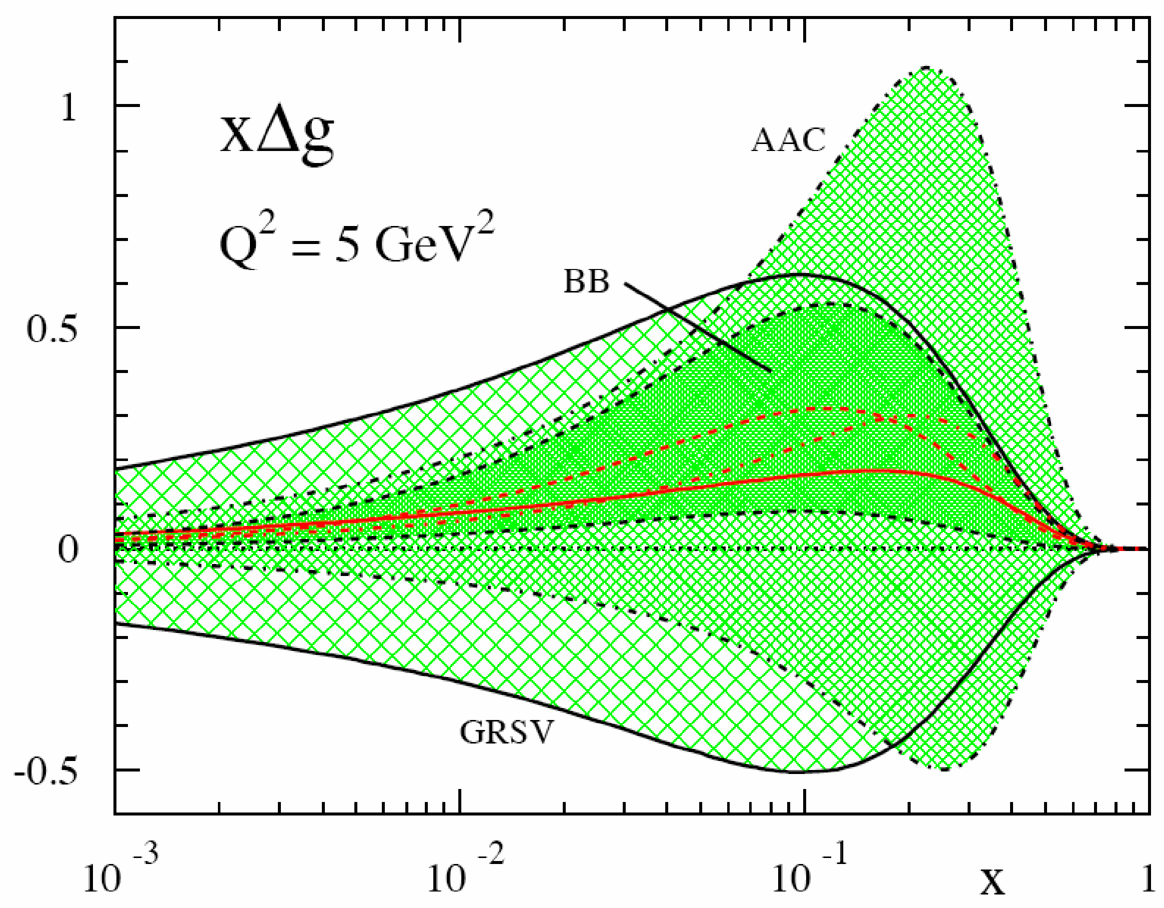
Quark spin ≈ 0.1

$$\Delta G = \int_0^1 dx\, \Delta g(x)$$

$\Delta g(x) =$  $-$ 

→ Much current activity of the field

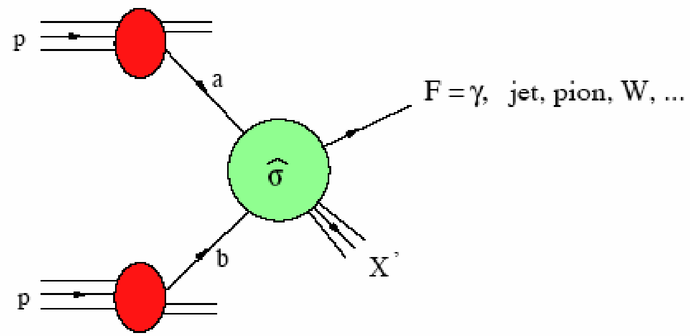
See http://spin.riken.bnl.gov/rsc/report/masterspin.pdf



Measurement of Δg a major emphasis at RHIC

Hard scattering in hadron collisions

$$p_T^3 \frac{d\sigma}{dp_T\, d\eta} = \left| \quad \right|^2 + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

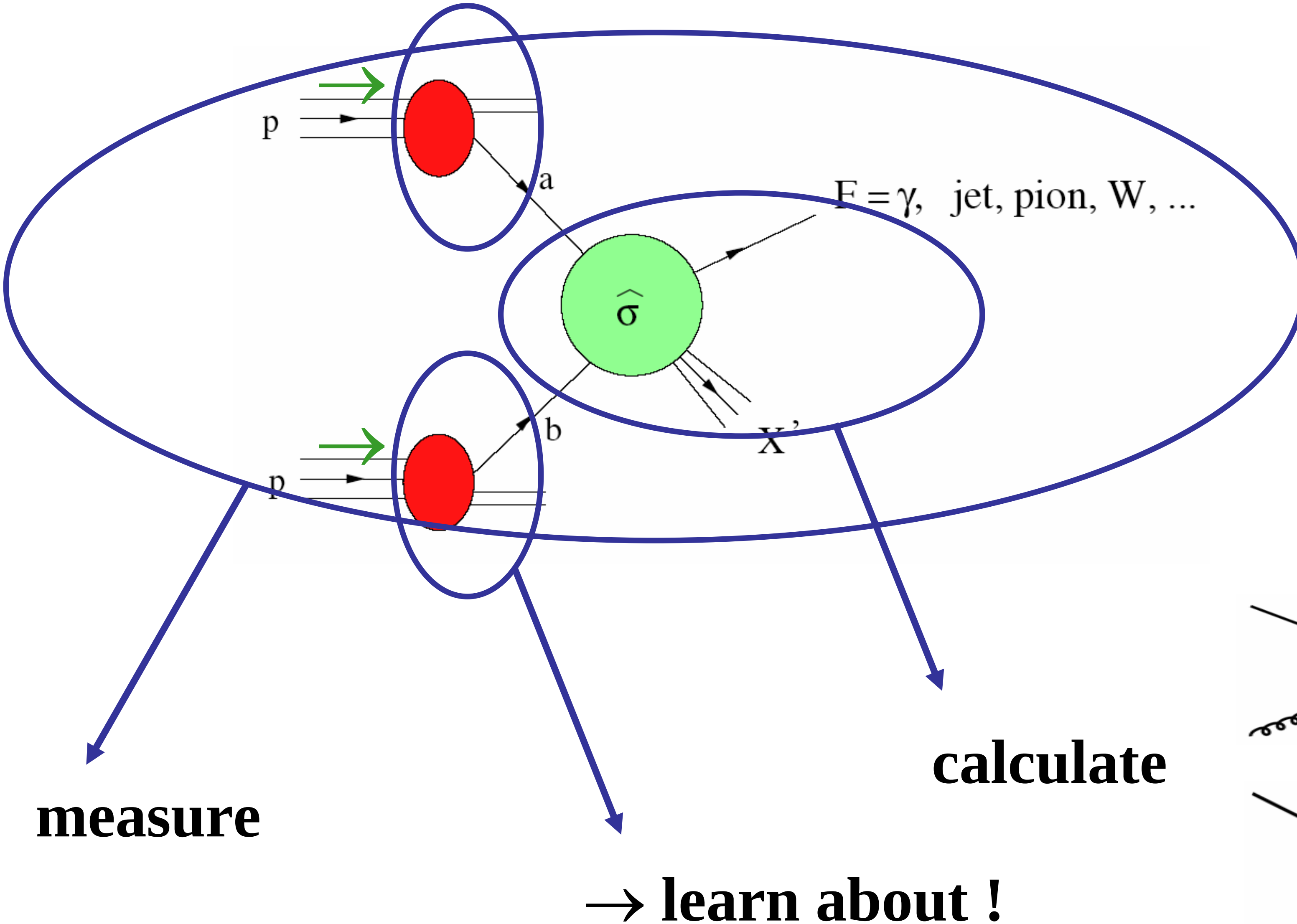


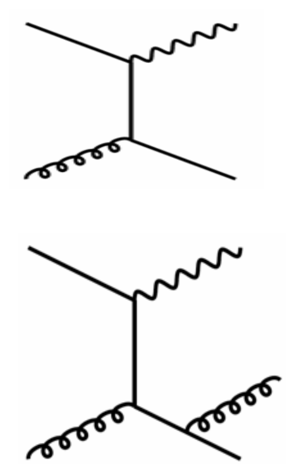
$$p_T^3 \frac{d\sigma^{pp \to \mathrm{F}X}}{dp_T\, d\eta} = \sum_{abc} \int dx_a\, dx_b\, f_a(x_a, \mu)\, f_b(x_b, \mu)$$

$$\times\, p_T^3 \frac{d\hat{\sigma}^{ab \to \mathrm{F}X'}}{dp_T\, d\eta}(x_a P_a, x_b P_b, P^{\mathrm{F}}, \mu) \;+\; \text{P.C.}$$

$$\hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \ldots \qquad \text{perturb.}$$

(for pions, additional fragmentation functions)



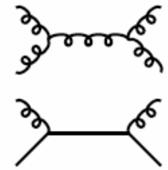
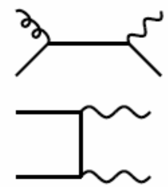
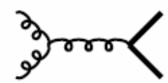
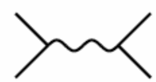
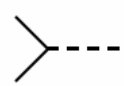


Needs: precision → higher orders in QCD perturbation th.

The last step is not trivial .

II. NLO predictions for RHIC

RHIC offers good possibilities to probe Δg :

Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	
$\vec{p}\vec{p} \to \mathrm{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)
$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \mathrm{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg	
$\vec{p}\vec{p} \to \mu^+\mu^- X$ (Drell-Yan) [78, 79, 80]	$\vec{q}\vec{\bar{q}} \to \gamma^* \to \mu^+\mu^-$	$\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to (Z^0, W^\pm)X$ $p\vec{p} \to (Z^0, W^\pm)X$ [78]	$\vec{q}\vec{\bar{q}} \to Z^0$, $\vec{q}'\vec{\bar{q}} \to W^\pm$ $\vec{q}'\bar{q} \to W^\pm$, $q'\vec{\bar{q}} \to W^\pm$	$\Delta q, \Delta\bar{q}$	

Jäger,Schäfer, Stratmann,WV

Jäger,Stratmann,WV; Signer et al.

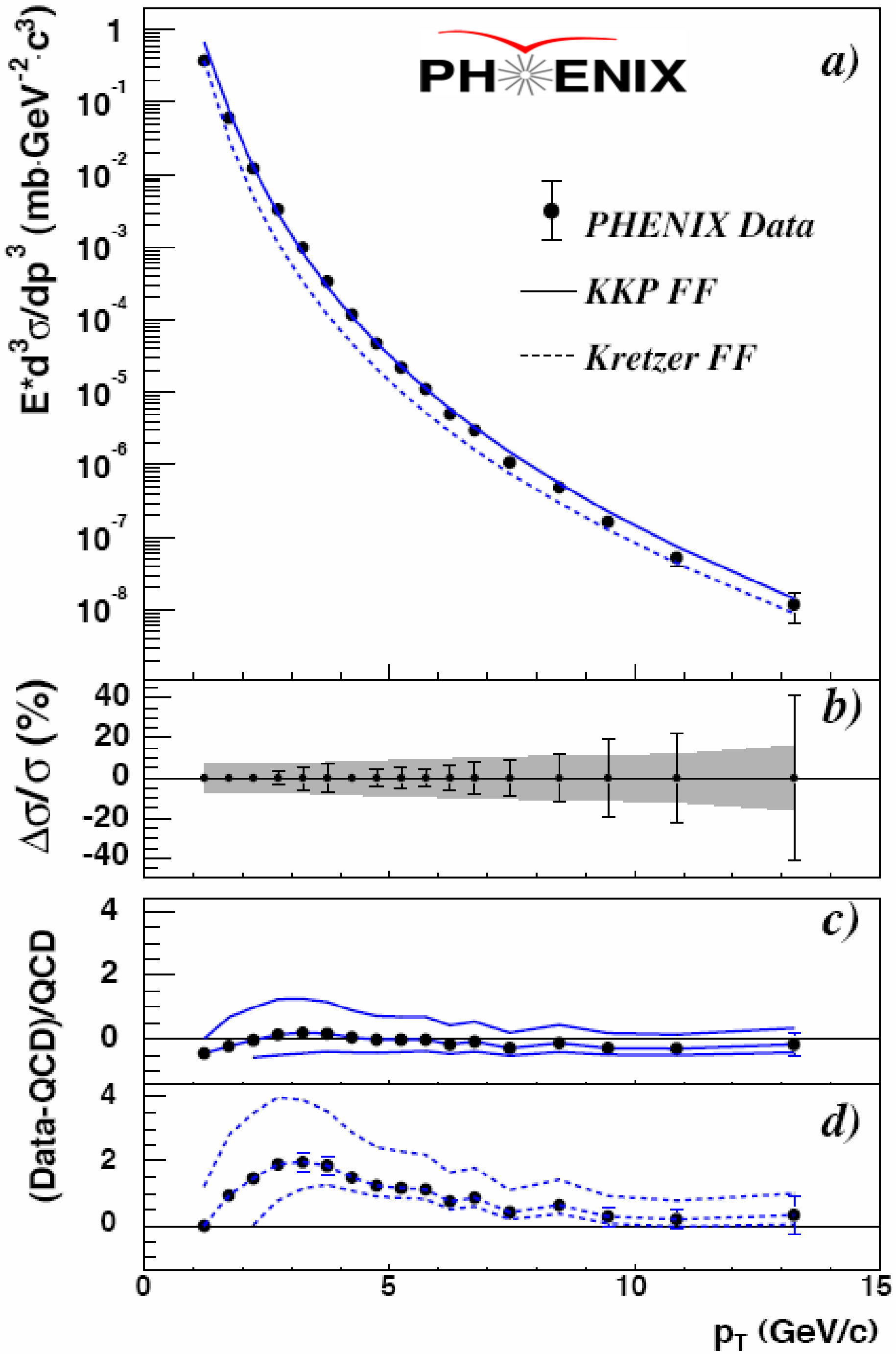
Gordon,WV; Contogouris et al.; Gordon, Coriano

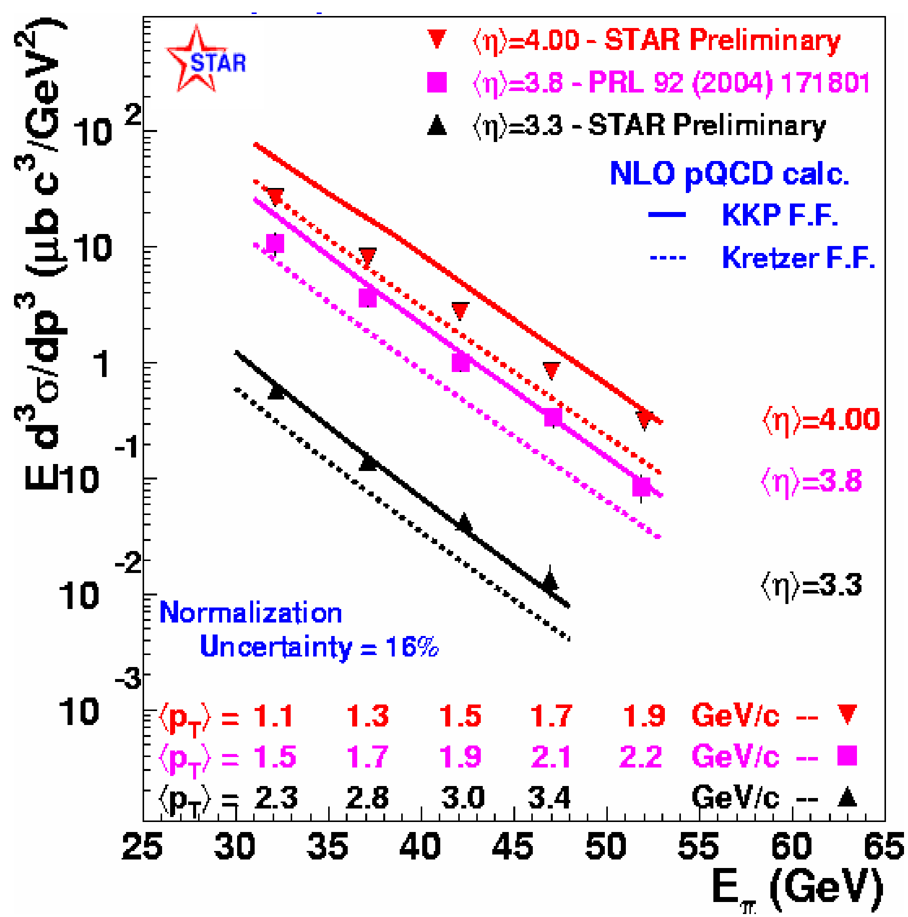
Stratmann, Bojak

Weber; Gehrmann; Kamal

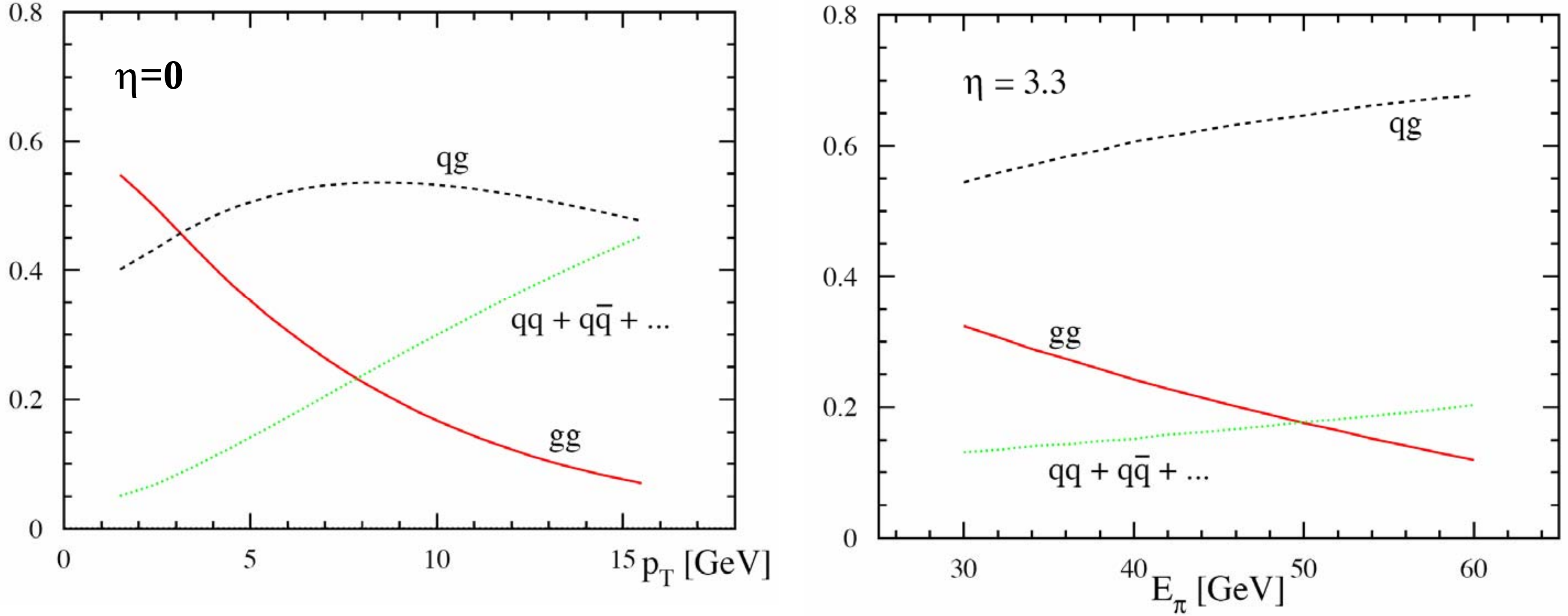
NLO corrections known in all cases .

$pp \rightarrow \pi^0 X$ at **RHIC**





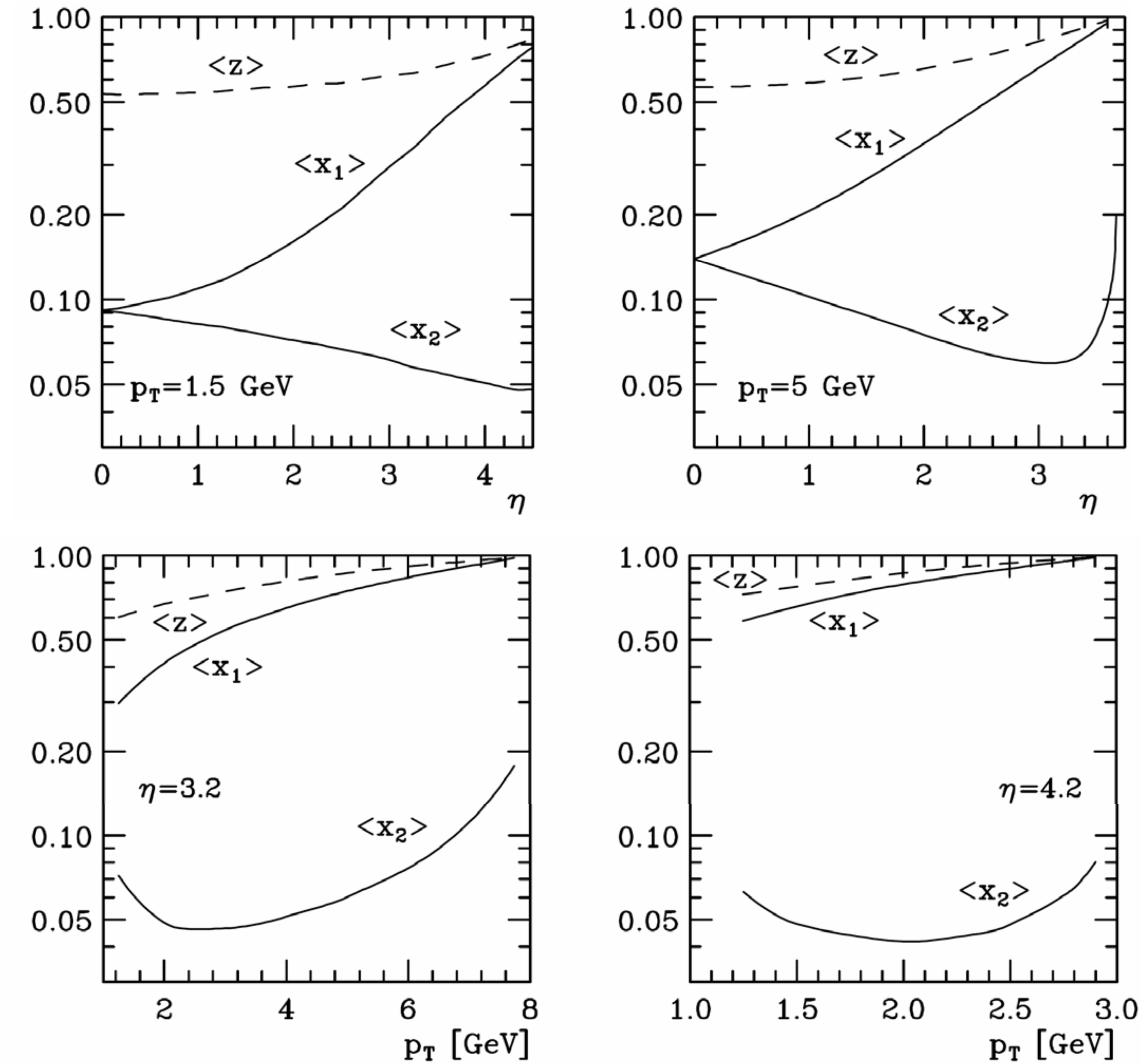
Contributions by subprocesses



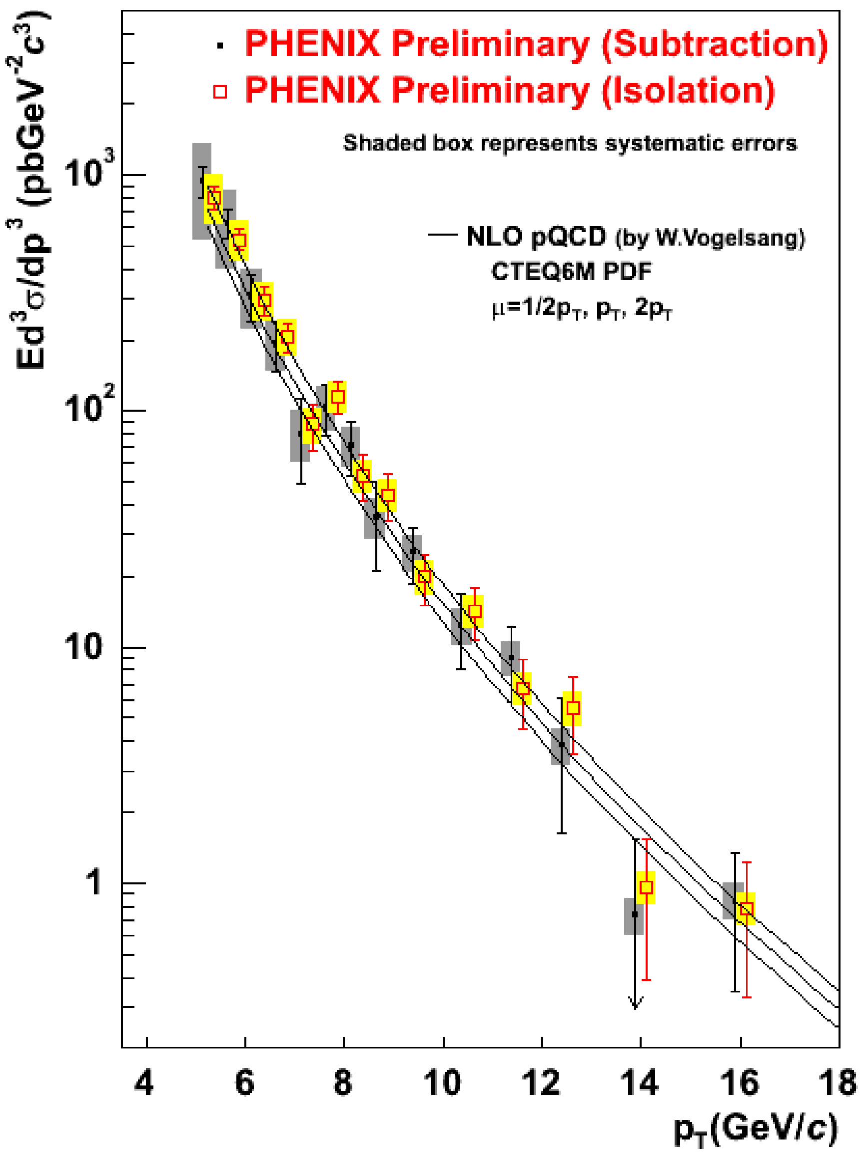
Average momentum fractions

(from Guzey, Strikman ,WV)

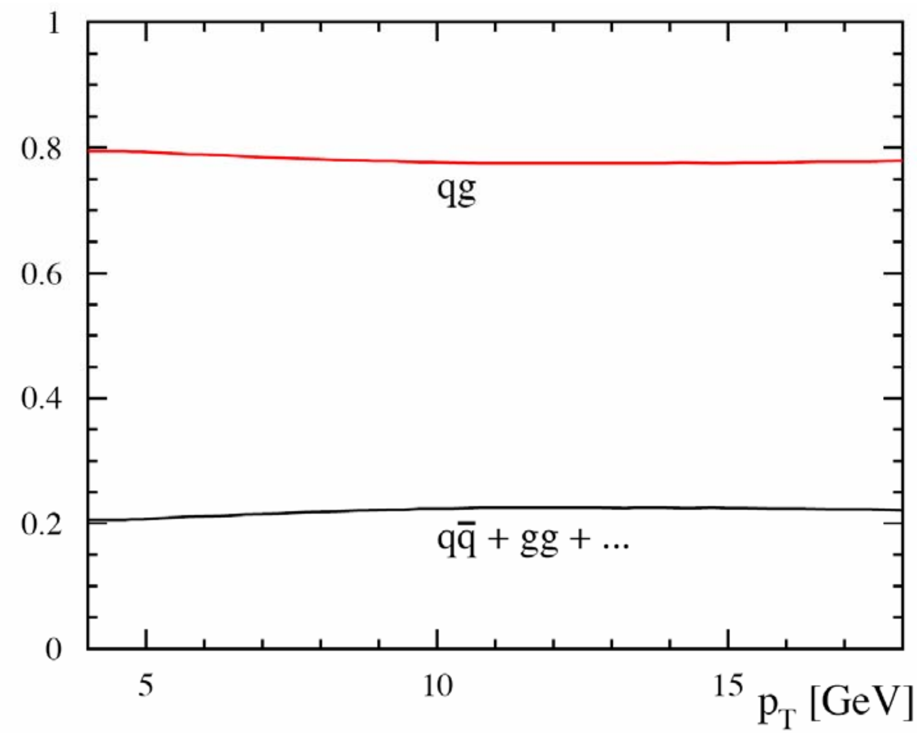
(S. Kretzer)



$pp \to \gamma X$

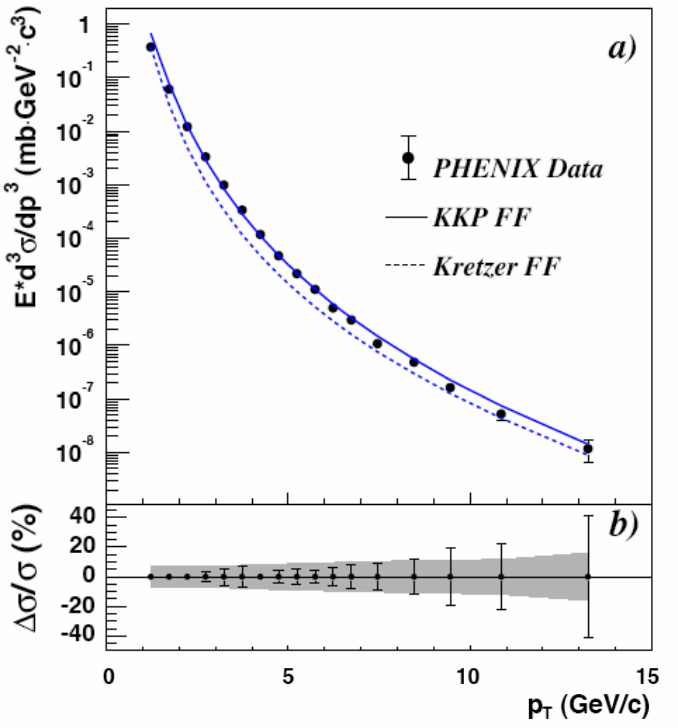


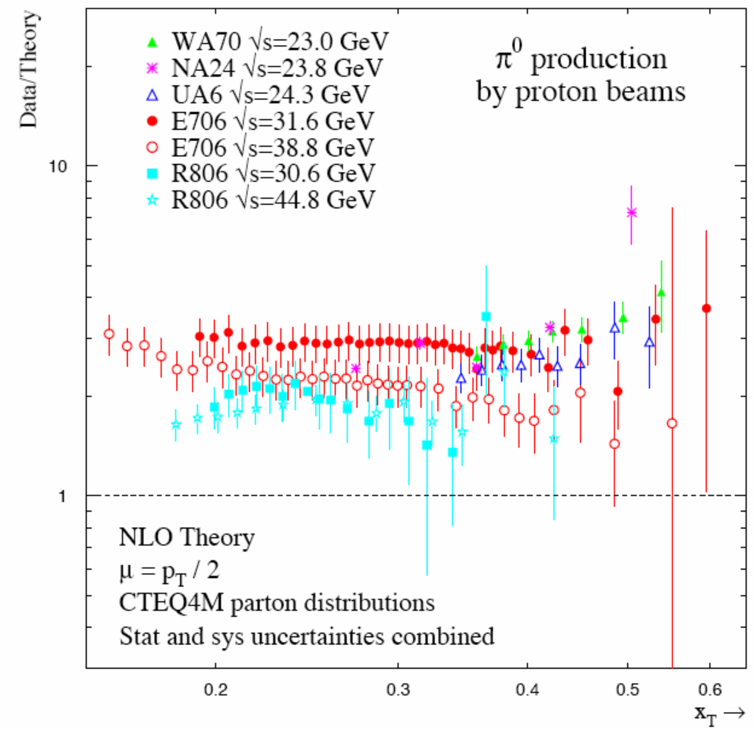
PHENIX Preliminary (Subtraction)
PHENIX Preliminary (Isolation)
Shaded box represents systematic errors
NLO pQCD (by W.Vogelsang)
CTEQ6M PDF
μ=1/2p_T, p_T, 2p_T
Ed^3σ/dp^3 (pbGeV^-2c^3)
p_T(GeV/c)



qg
$q\bar{q}$ + gg + ...
p_T [GeV]

• **A long-standing problem :** $pp \rightarrow \pi^0 X$





…well described by NLO at RHIC

…but data much higher than NLO at fixed-target energies !

• **Detailed understanding of this issue is also important for spin physics at RHIC**

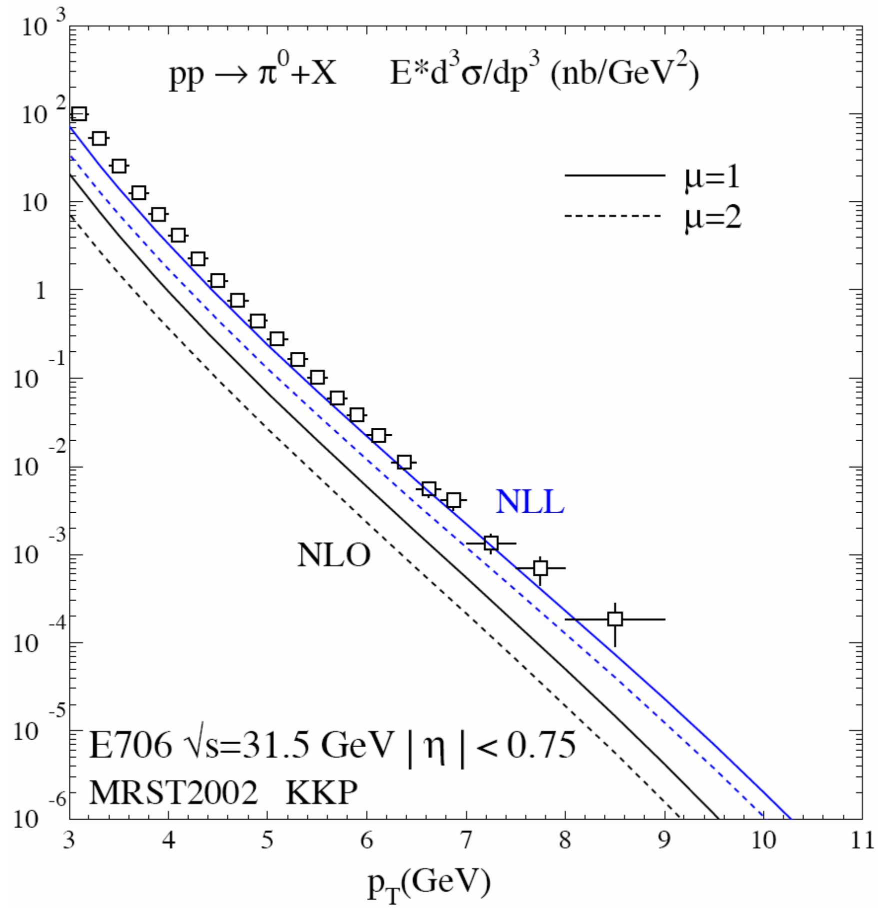
- **Resummation of important higher-order corrections beyond NLO** **de Florian, WV**

$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\text{Born}}}{dp_T} \Bigg[1 + \underbrace{\mathcal{A}_1\, \alpha_s \ln^2\left(1-\hat{x}_T^2\right) + \mathcal{B}_1\, \alpha_s \ln\left(1-\hat{x}_T^2\right)}_{\text{NLO}}$$

$$+ \ldots + \mathcal{A}_k\, \alpha_s^k \ln^{2k}\left(1-\hat{x}_T^2\right) + \ldots \Bigg]$$

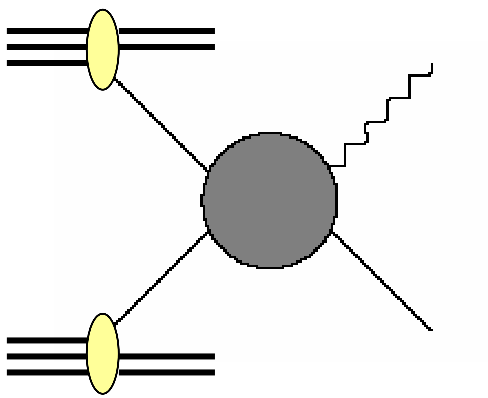
$$\hat{x}_T \equiv \frac{2p_T}{\sqrt{\hat{s}}}$$

new terms lead to strong enhancement &
improvement of theory vs. data →



Application to prompt photons: $pp \to \gamma X$

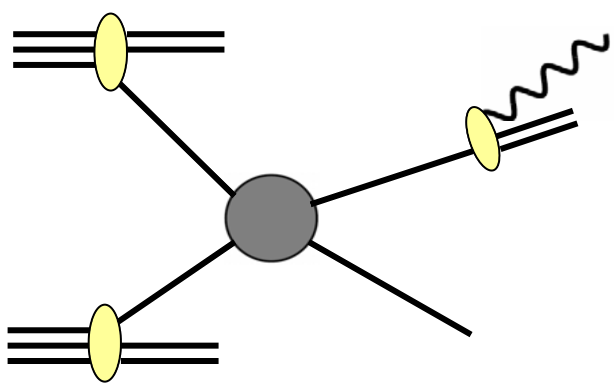
"direct" contributions:



relatively small resum. effects

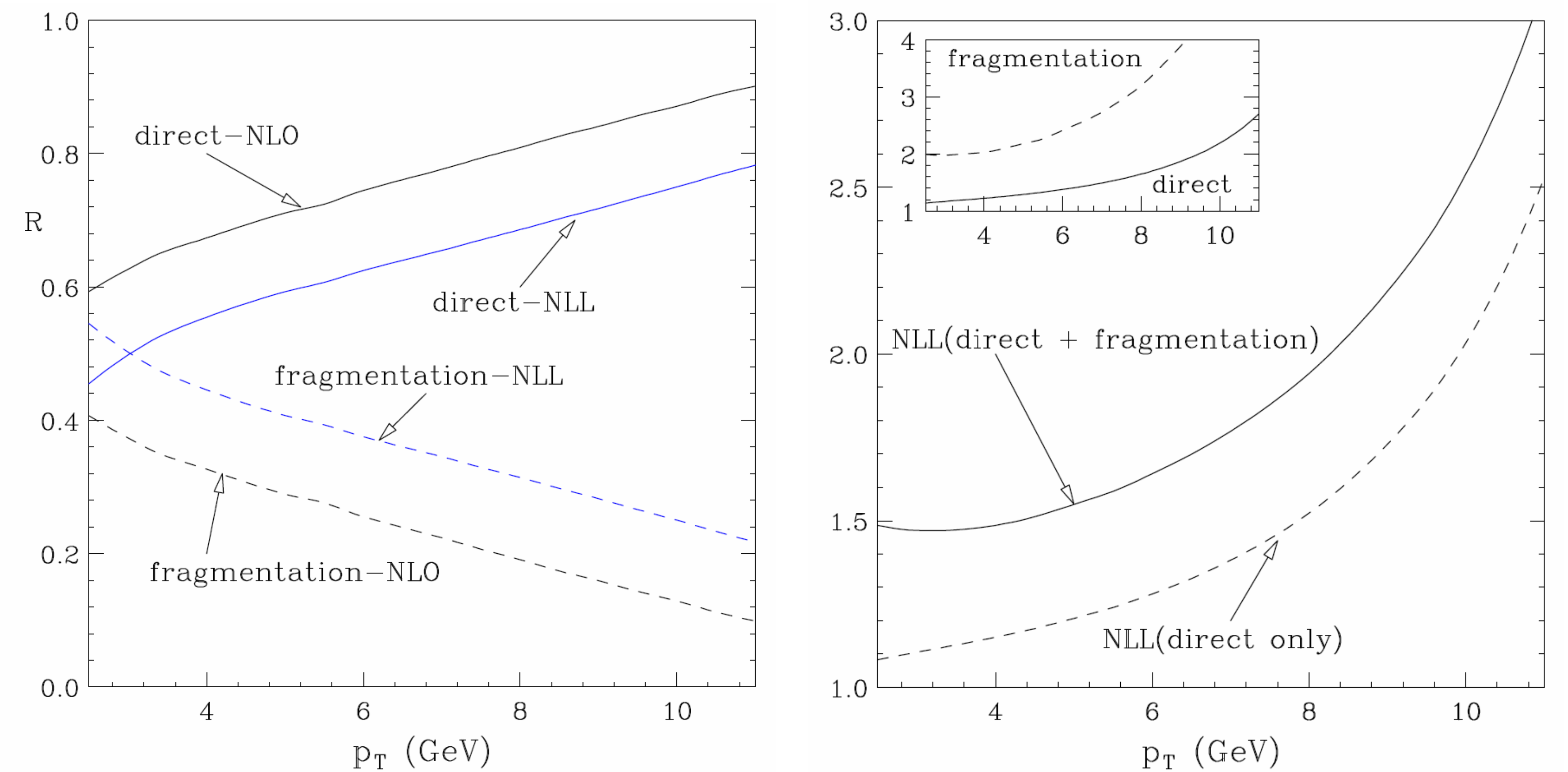
(Catani et al.; Sterman, WV; Kidonakis, Owens)

"fragmentation" contributions:

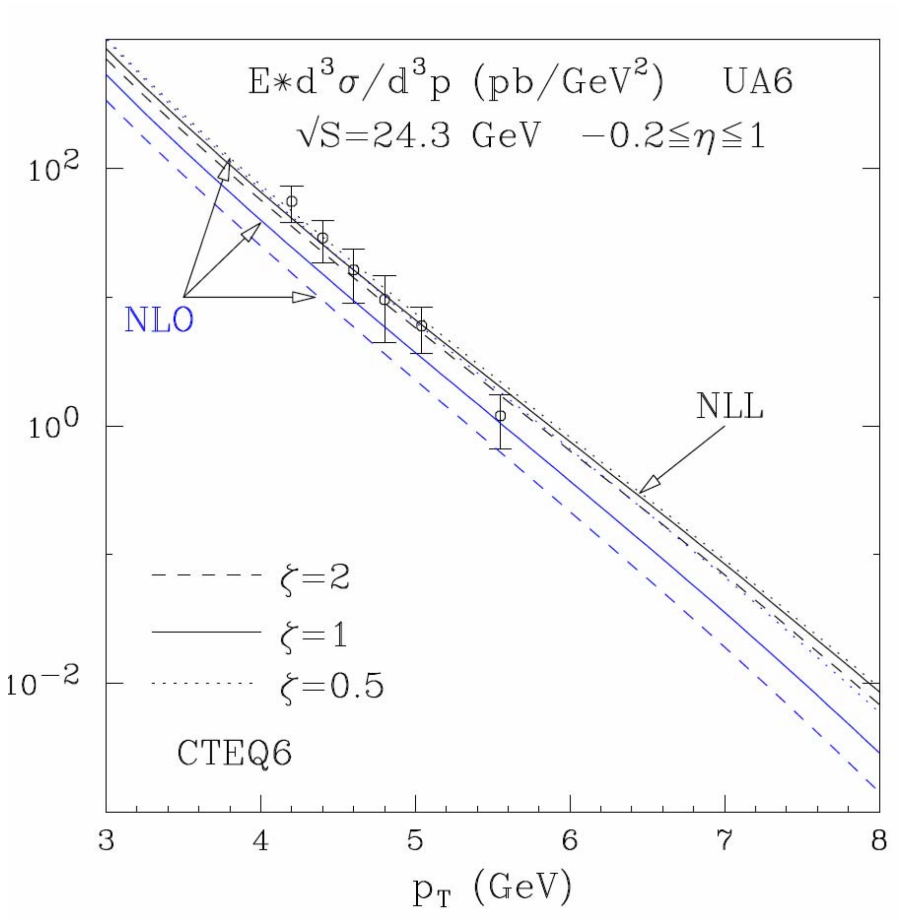


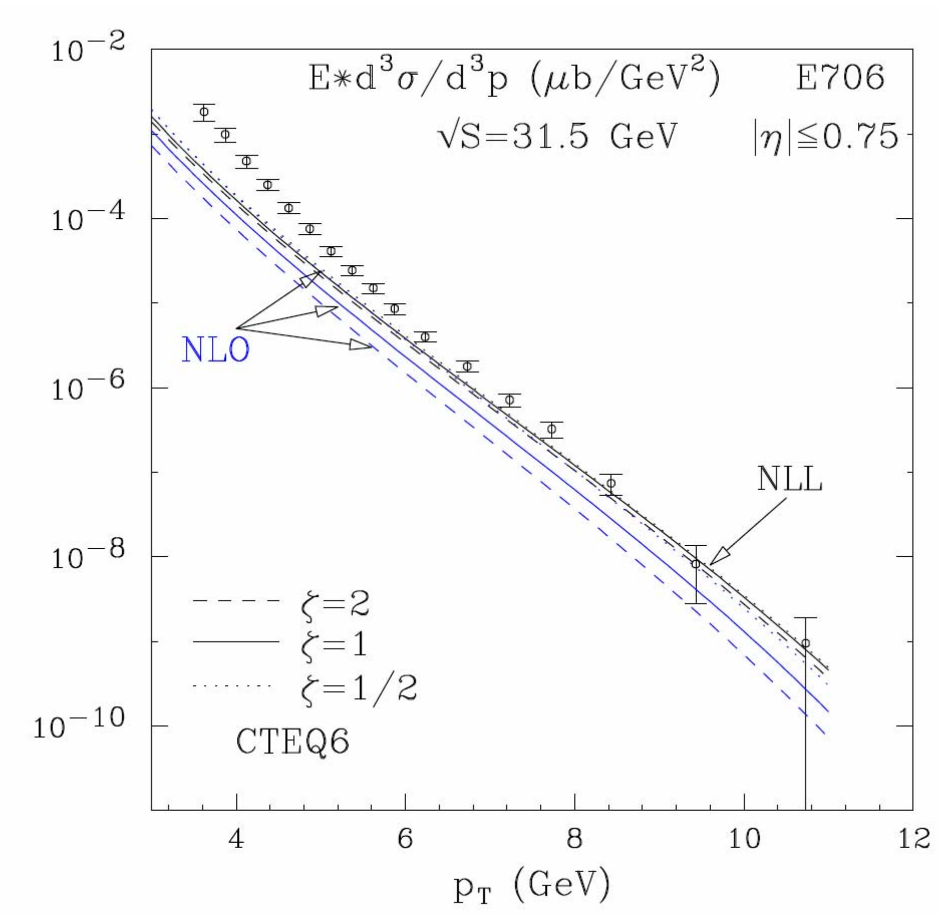
a bit like π^0 production, but less $gg \to gg$ because D_g^γ is smaller

de Florian, WV



de Florian, WV





Now to A_{LL} and Δg

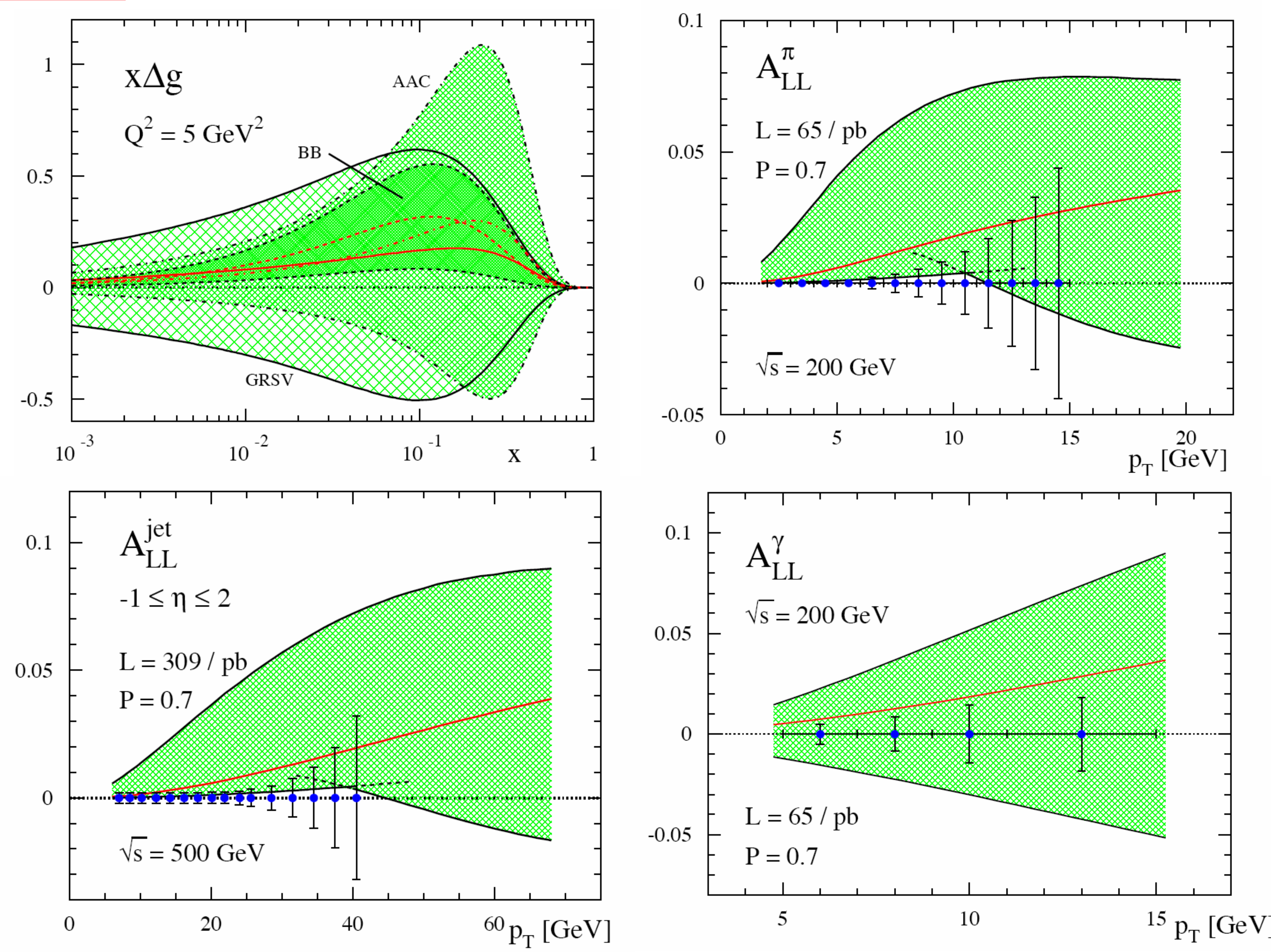
- convolutions "pdf $\times$ pdf $\times$ cr.sec." relatively complicated. "inversion" $A_{LL} \rightarrow \Delta g(x)$ in general not straightforward

- at the moment emphasis is on NLO predictions of spin asymmetries in terms of "model" Δg, to study the sensitivity of the observable

- AAC (Hirai, Kumano, Saito) have made first attempts to estimate impact of future RHIC data

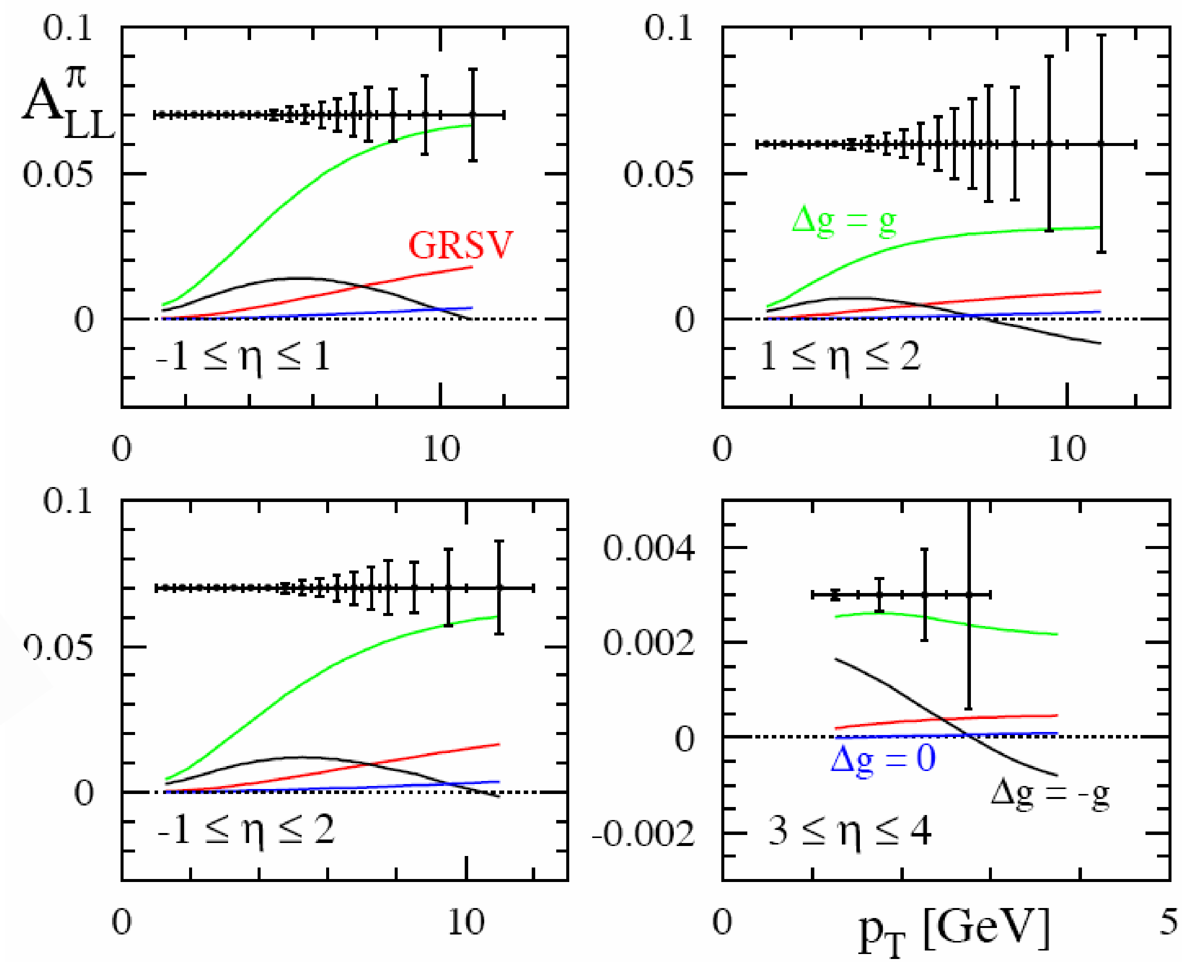
- future: CTEQ-style "global" analyis of variety of A_{LL} data. Should inlcude NLO.

- alternative approach: "correlations" (γ+jet, $\pi+\pi$) that probe kinematics in more detail



xΔg
Q² = 5 GeV²
AAC
BB
GRSV
x
A^π_LL
L = 65 / pb
P = 0.7
√s = 200 GeV
p_T [GeV]
A^jet_LL
-1 ≤ η ≤ 2
L = 309 / pb
P = 0.7
√s = 500 GeV
p_T [GeV]
A^γ_LL
√s = 200 GeV
L = 65 / pb
P = 0.7
p_T [GeV]

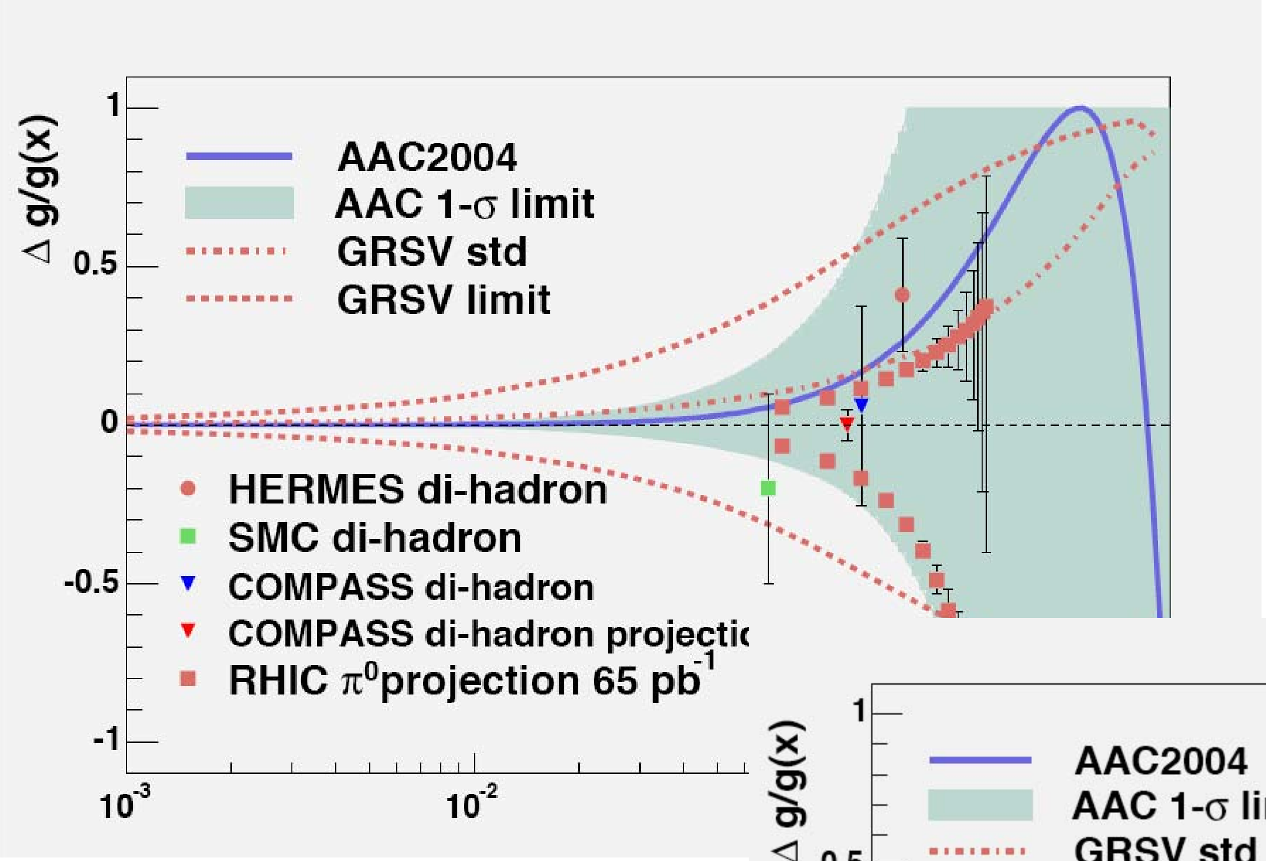
Rapidity dependence of A_{LL}



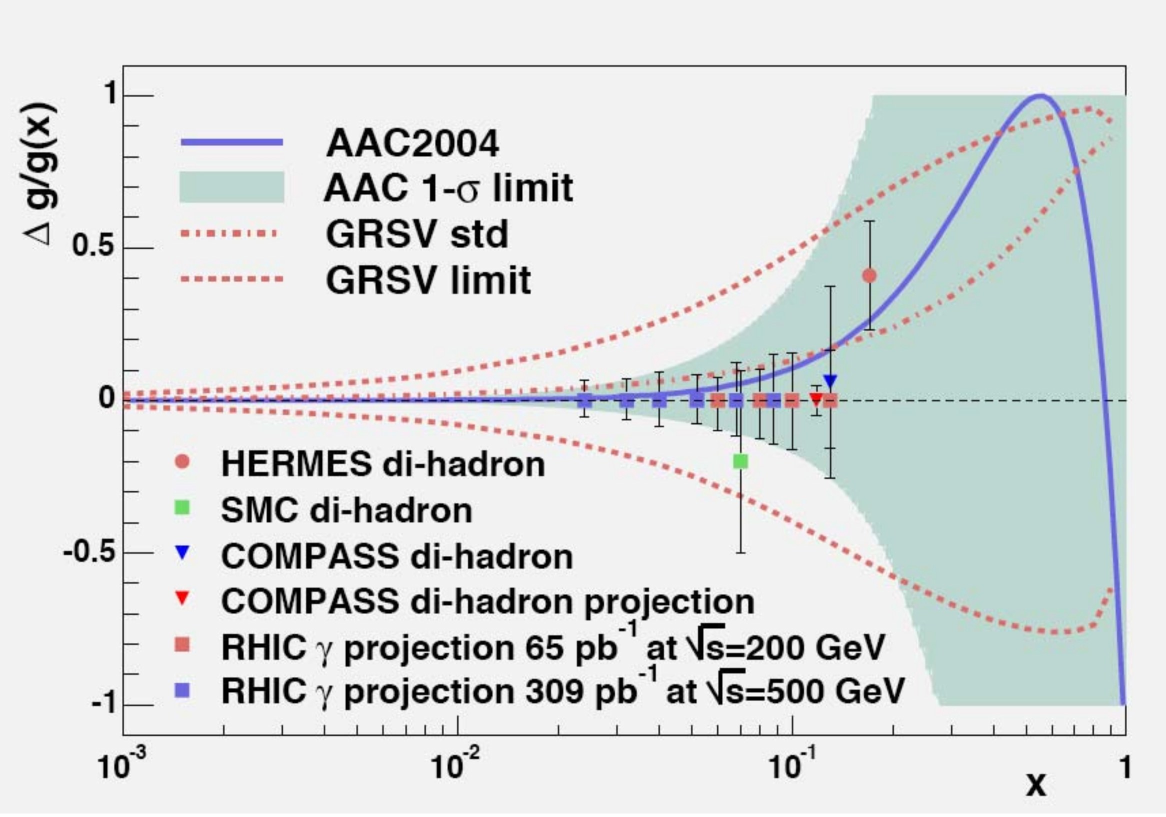
(Stratmann, Jäger, WV)

(L = 7/pb, P = 0.4)

N. Saito



$\Delta g/g(x)$
AAC2004
AAC 1-σ limit
GRSV std
GRSV limit
HERMES di-hadron
SMC di-hadron
COMPASS di-hadron
COMPASS di-hadron projectic
RHIC π^0projection 65 pb^{-1}
1
0.5
0
-0.5
-1
10^{-3}
10^{-2}

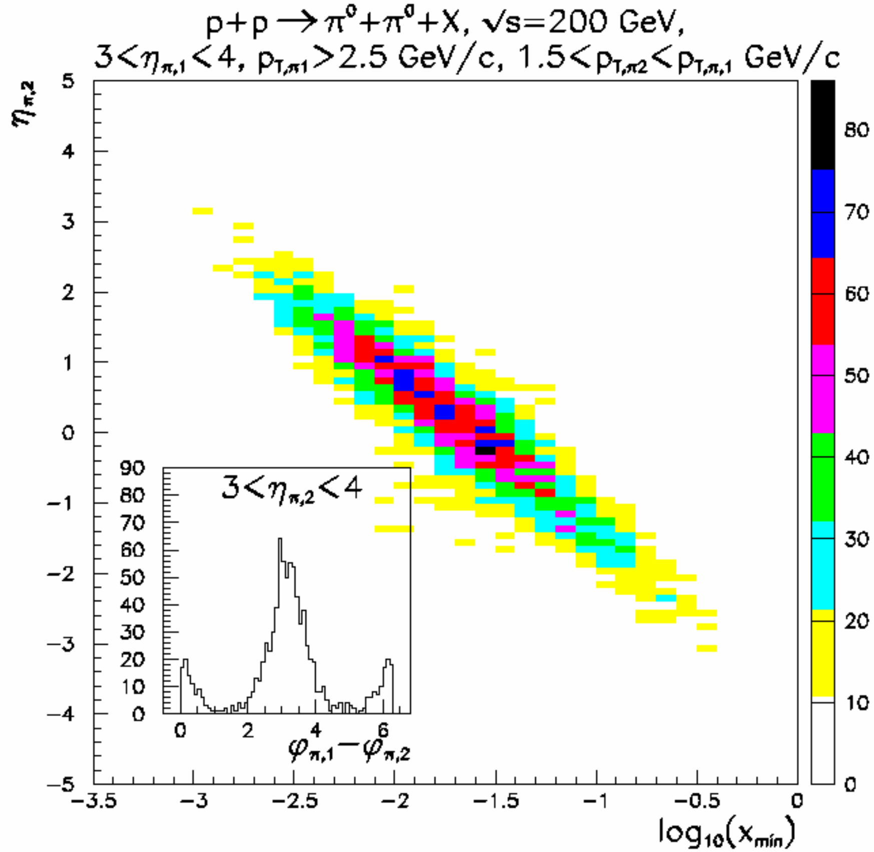


$\Delta g/g(x)$
AAC2004
AAC 1-σ limit
GRSV std
GRSV limit
HERMES di-hadron
SMC di-hadron
COMPASS di-hadron
COMPASS di-hadron projection
RHIC γ projection 65 pb^{-1} at $\sqrt{s}$=200 GeV
RHIC γ projection 309 pb^{-1} at $\sqrt{s}$=500 GeV
1
0.5
0
-0.5
-1
10^{-3}
10^{-2}
10^{-1}
1
x

- "correlations", e.g. in pp $\rightarrow$ photon+jet+X

@ LO $\quad x_1 = \frac{p_T}{\sqrt{s}} \left(e^{-\eta_1} + e^{-\eta_1}\right) \qquad x_2 = \frac{p_T}{\sqrt{s}} \left(e^{\eta_1} + e^{\eta_2}\right)$

- recent study for pp $\rightarrow \pi\pi X$: L. Bland



- can "dial" x_2
- NLO available in unpol. case (Owens; Binoth, Guillet, Pilon, Werlen)
- pol. soon … Jäger, Owens, Stratmann, WV
- some of the advantage will be lost, but sensitivity is there
- in any case, good tool for direct information, small-x

III. Technique for "global analysis"

Prospects of learning from data
relies on our ability to efficiently evaluate

$$\sigma = \sum_{a,b} f_a \otimes f_b \otimes \hat{\sigma}_{ab}$$

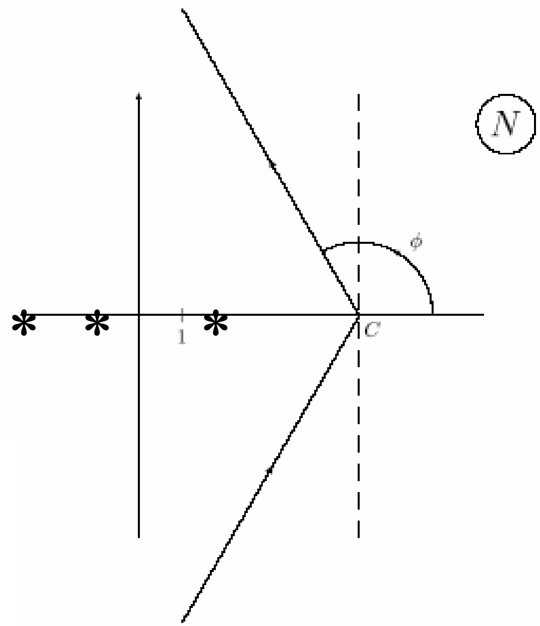
- need "global analysis"
  - input pdfs at scale μ_0 in terms of ansatz with free parameters
  - evolve to scale μ relevant to a data point
  - compare to data and assign χ^2 value
  - vary parameters and minimize χ^2

- requires typically 1000's of evaluations of the cross section

- want $\hat{\sigma}_{ab}$ at order "as high as possible"
  - theoretical uncertainties decrease
  - but already NLO often numerically involved and time-consuming

Moments of a function $f(x)$:

$$f^{\,n} \;\equiv\; \int_0^1 dx\; x^{n-1}\, f(x)$$



The Mellin inverse :

$$f(x) = \frac{1}{2\pi i}\int_{C_n} dn\; x^{-n}\, f^{\,n}$$

A property : for a convolution,

$$\big(f \otimes g\big)(x) \;\equiv\; \int_x^1 \frac{dz}{z}\, f(z)\, g\left(\frac{x}{z}\right)$$

$$\boxed{\big(f \otimes g\big)^n \;=\; f^{\,n}\, g^{\,n}}$$

Well-known applications:

- DGLAP evolution of pdfs. For example,

$$\frac{\mathrm{d}\, q(x,\mu)}{\mathrm{d} \log \mu^2} = P_{qq} \otimes q + P_{qg} \otimes g \ ,$$

therefore,

$$\frac{\mathrm{d}\, q^n(\mu)}{\mathrm{d} \log \mu^2} = P^n_{qq}\, q^n + P^n_{qg}\, g^n$$

- DIS structure functions :

$$\int_0^1 dx\, x^{n-1} g_1(x,Q^2) = \frac{1}{2} \sum_q C^n_q \left(\Delta q^n(Q^2) + \Delta \bar{q}^n(Q^2)\right) + \ldots$$

Makes Mellin moments ideal for analyses of DIS

- wide class of hadron-hadron cross sections : single-particle inclusive (or even less incl.) for instance, *prompt photons* at RHIC:

$$\frac{d\sigma^\gamma}{dp_T d\eta} = \sum_{a,b} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 dx_b \; f_a(x_a,\mu)\, f_b(x_b,\mu)\, \frac{d\hat{\sigma}^\gamma_{ab}}{dp_T d\eta}(\hat{s}\,, p_T, \eta,\, \mu)$$

$$x_a^{\min} = x_T\, \mathrm{e}^{\eta}/(2 - x_T\, \mathrm{e}^{-\eta})$$

$$x_b^{\min} = x_a\, x_T\, \mathrm{e}^{-\eta}/(2\, x_a - x_T\, \mathrm{e}^{\eta}) \qquad (x_T = 2p_T/\sqrt{S}\ )$$

- only after integration over *all* η :

$$\frac{d\sigma^\gamma}{dp_T} = \sum_{a,b} \int_{x_T^2}^1 dx_a \int_{x_T^2/x_a}^1 dx_b\, f_a(x_a,\mu)\, f_b(x_b,\mu)\, \frac{d\hat{\sigma}^\gamma_{ab}}{dp_T}(\hat{x}_T^2 = x_T^2/x_a x_b, \mu/p_T)$$

factorizes under Mellin moments,

$$\int_0^1 dx_T^2\, \left(x_T^2\right)^{n-1} p_T^3\, \frac{d\sigma^\gamma}{dp_T} = f_a^{\,n+1}\, f_b^{\,n+1}\, \left(\hat{\sigma}^\gamma_{ab}\right)^n$$

$$\left(\hat{\sigma}^\gamma_{ab}\right)^n \equiv \int_0^1 d\hat{x}_T^2\, \left(\hat{x}_T^2\right)^{n-1} p_T^3\, \frac{d\hat{\sigma}^\gamma_{ab}}{dp_T}(\hat{x}_T^2)$$

General Mellin technique

Stratmann,WV

earlier ideas : Berger,Graudenz,Hampel,Vogt; Kosower

Consider general cross sec. for producing final state H with observed variable O

$$\frac{d\sigma^H}{dO} = \sum_{a,b,c} \int_{\mathrm{exp-bin}} dT \int_{x_a^{\mathrm{min}}}^1 dx_a \int_{x_b^{\mathrm{min}}}^1 dx_b \int_{z_c^{\mathrm{min}}}^1 dz_c$$

$$\times\, f_a(x_a,\mu_F)\, f_b(x_b,\mu_F)\, D_c^H(z_c,\mu_F')$$

LO + NLO + …

$$\times\, \frac{d\hat{\sigma}_{ab}^c}{dO\,dT}(x_a P_A, x_b P_B, P_H/z_c, T, \mu_R, \mu_F, \mu_F')\ ,$$

could be *any*thing : $\vec{p}\vec{p} \to \gamma X$, $ep \to \mathrm{jet} + X$, …

Express pdfs by their Mellin inverses :

$$f_a(x_a,\mu_F) = \frac{1}{2\pi i}\int_{\mathcal{C}_n} dn\; x_a^{-n}\, f_a^{\,n}(\mu_F)$$

$$f_b(x_b,\mu_F) = \frac{1}{2\pi i}\int_{\mathcal{C}_m} dm\; x_b^{-m}\, f_b^{\,m}(\mu_F)$$

Treated as known here. Method could be used to determine ff's: Kretzer, Yokoya

$$\frac{d\sigma^H}{dO} = \frac{1}{(2\pi i)^2}\sum_{a,b,c}\int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm\; f_a^{\,n}(\mu_F)\, f_b^{\,m}(\mu_F)$$

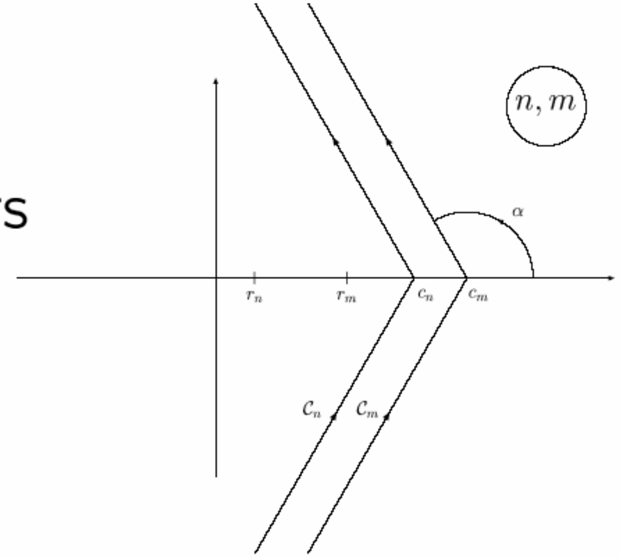
$$\times \int_{\text{exp-bin}} dT \int_{x_a^{\text{min}}}^{1} dx_a \int_{x_b^{\text{min}}}^{1} dx_b \int_{z_c^{\text{min}}}^{1} dz_c\; x_a^{-n} x_b^{-m}\; D_c^H(z_c,\mu_F')$$

$$\times \frac{d\hat{\sigma}_{ab}^c}{dO\,dT}(x_a P_A,\, x_b P_B\,,\, P_H/z_c, T,\, \mu_R, \mu_F, \mu_F')$$

$$\equiv \sum_{a,b}\int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm\; f_a^{\,n}(\mu_F)\, f_b^{\,m}(\mu_F)\; \tilde{\sigma}_{ab}^H(n,m,O,\mu_R,\mu_F)$$

$$\sum_{a,b} \int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm \; f_a^{\,n}(\mu_F)\, f_b^{\,m}(\mu_F)\; \tilde{\sigma}_{ab}^{H}(n,m,O,\mu_R,\mu_F)$$

- $\tilde{\sigma}_{ab}^{H}(n,m,O,\mu_R,\mu_F)$ is cross section for "dummy" pdfs $x_a^{-n} \times x_a^{-m}$
- all tedious integrations in $\tilde{\sigma}_{ab}^{H}(n,m,O,\mu_R,\mu_F)$
- can be pre-calculated on a suitable grid in n, m
- for optimal contours, *exponential decrease* of x_a^{-n}, x_a^{-m} along contours
- pdfs fall off at least as fast as $1/|n|^4$, $1/|m|^4$



Only *n,m* integrations left !

Toy analysis: prompt photons at RHIC

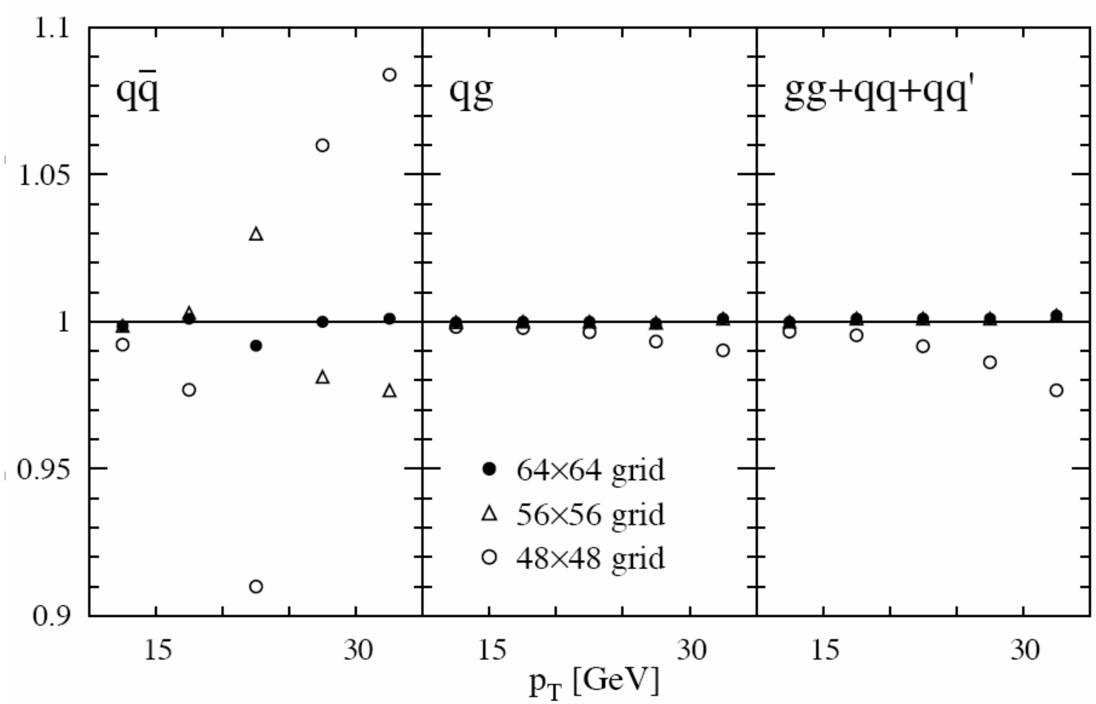
- NLO, scales $\mu_F = \mu_R = p_T$
- $\sqrt{S} = 200$ GeV, $|\eta| < 0.35$, isolated cr. sec.
- "ficticious" data points at
  $p_T = 12.5, 17.5, 22.5, 27.5, 32.5$ GeV
  calc. with GRSV $\oplus$ random Gaussian 1σ shift
- fit to DIS *and* prompt photon "data"
  ansatz for gluon density :

  $$\Delta g(x, \mu_0) = N\, x^{\alpha}\, (1-x)^{\beta}\, (1+\gamma\, x)\; g(x, \mu_0)$$

- perform large number of fits;
  allow for $\Delta\chi^2 = 4$ to obtain "error band"

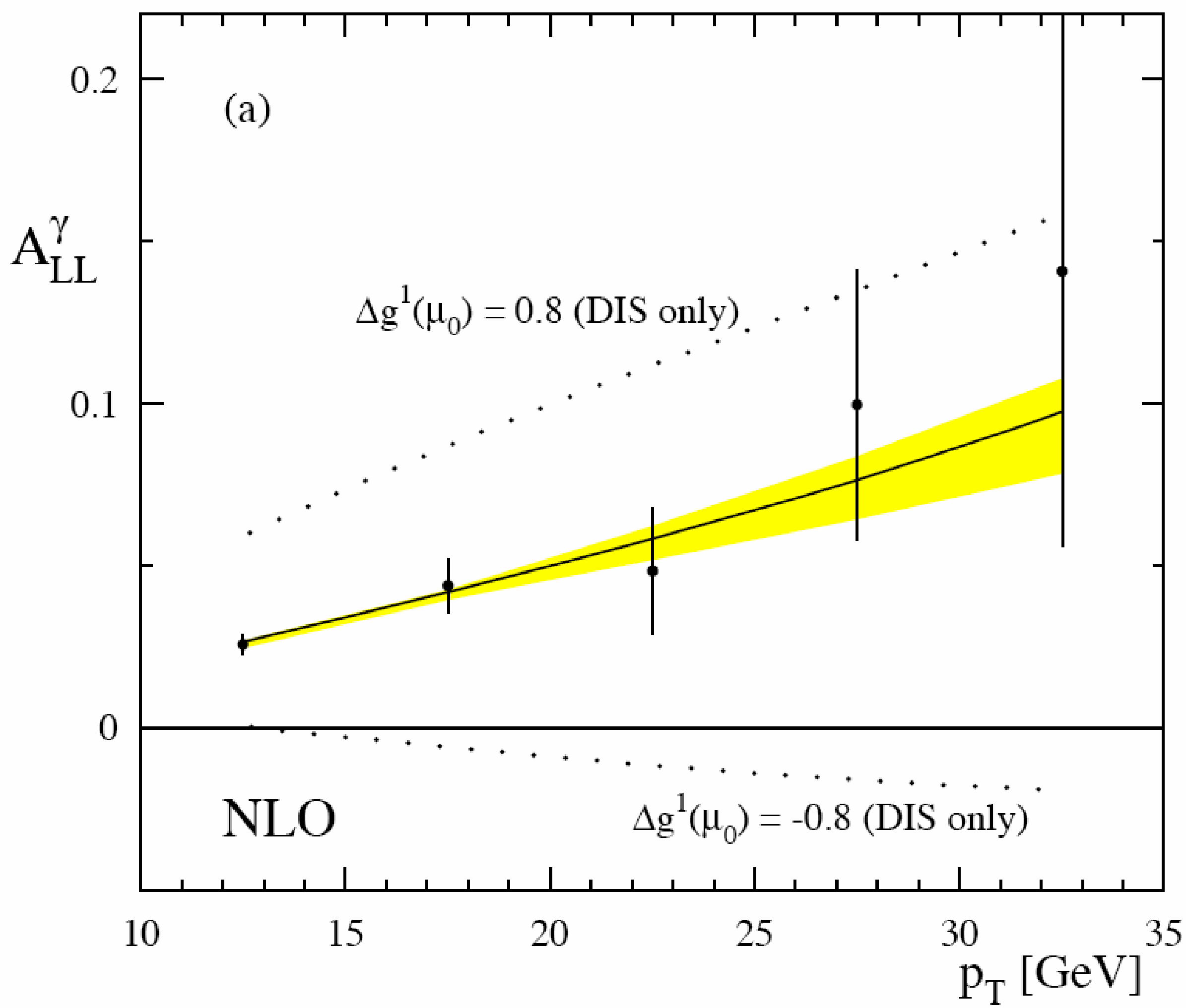
Accuracy of method:

Stratmann, WV

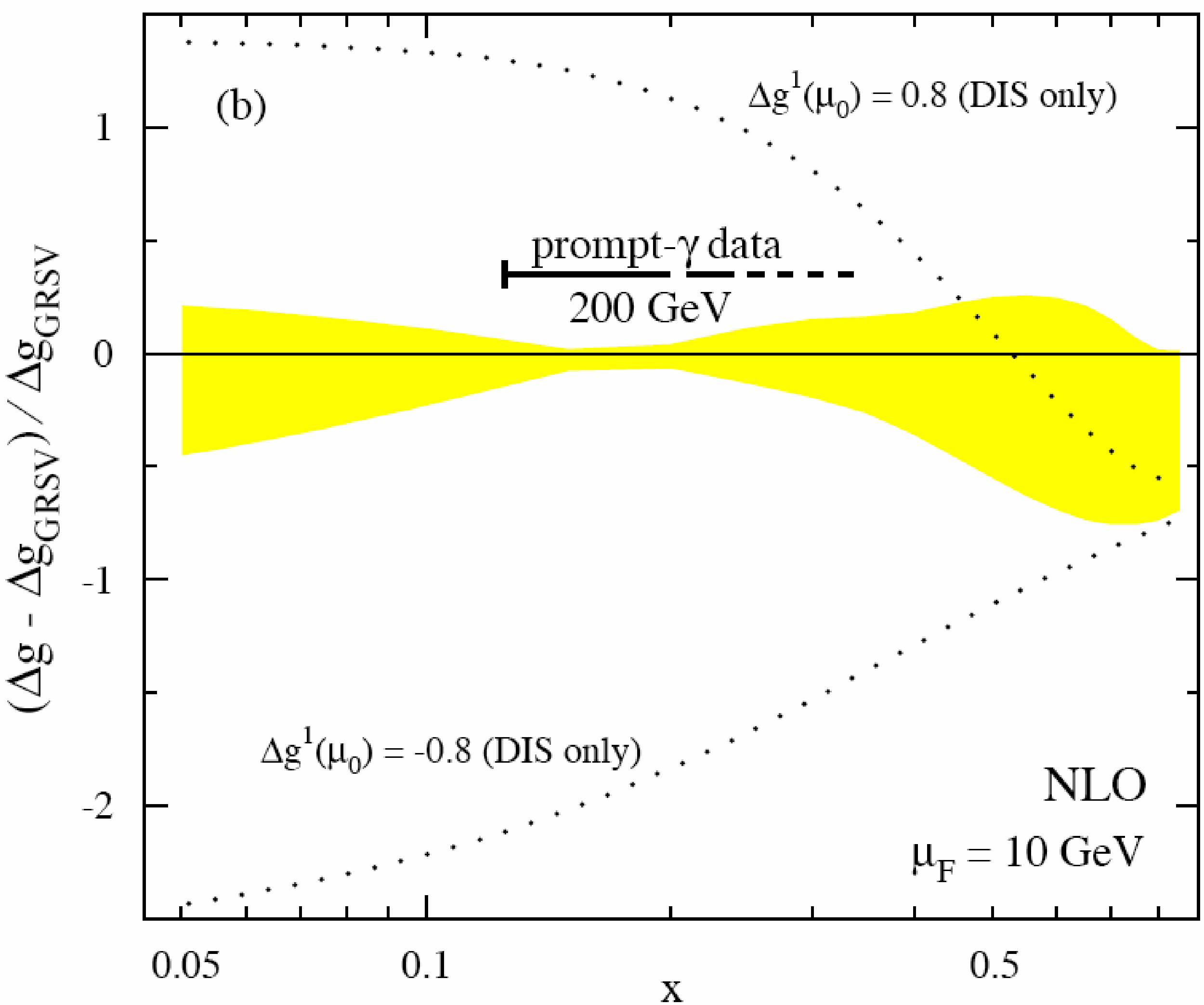


Evaluation of cross section extremely fast :

- generation of grids in n, m takes $\sim$ 5 hrs.
- after that :
  1000 evaluations of cr. sec. in $\sim$ 10 sec.



(a)
A_{LL}^{γ}
0.2
0.1
0
$\Delta g^1(\mu_0) = 0.8$ (DIS only)
$\Delta g^1(\mu_0) = -0.8$ (DIS only)
NLO
10
15
20
25
30
35
p_T [GeV]



(b)
$\Delta g^1(\mu_0) = 0.8$ (DIS only)
prompt-γ data
200 GeV
$\Delta g^1(\mu_0) = -0.8$ (DIS only)
NLO
$\mu_F = 10$ GeV
$(\Delta g - \Delta g_{GRSV}) / \Delta g_{GRSV}$
x
1
0
-1
-2
0.05
0.1
0.5

IV. Conclusions

- **have solid (and tested!) theoretical framework for calculations of A_{LL} at RHIC**

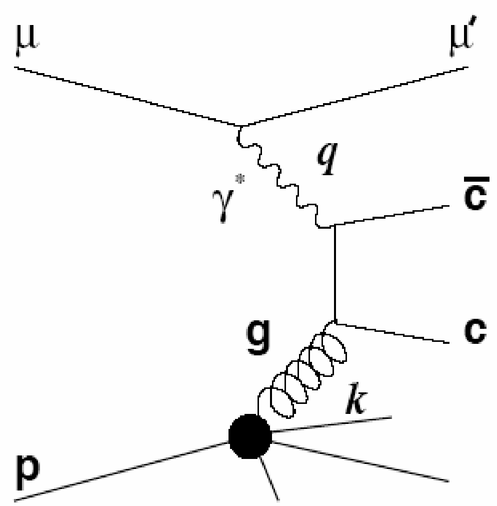
- **extraction of Δg will require simultaneous analysis of varied data sets**

- **still a lot of work remains to make "global analysis" work for polarized case**

• lepton-nucleon : $\gamma p \to c\bar{c}X$

Watson; Glück,Reya,WV

Bojak, Stratmann



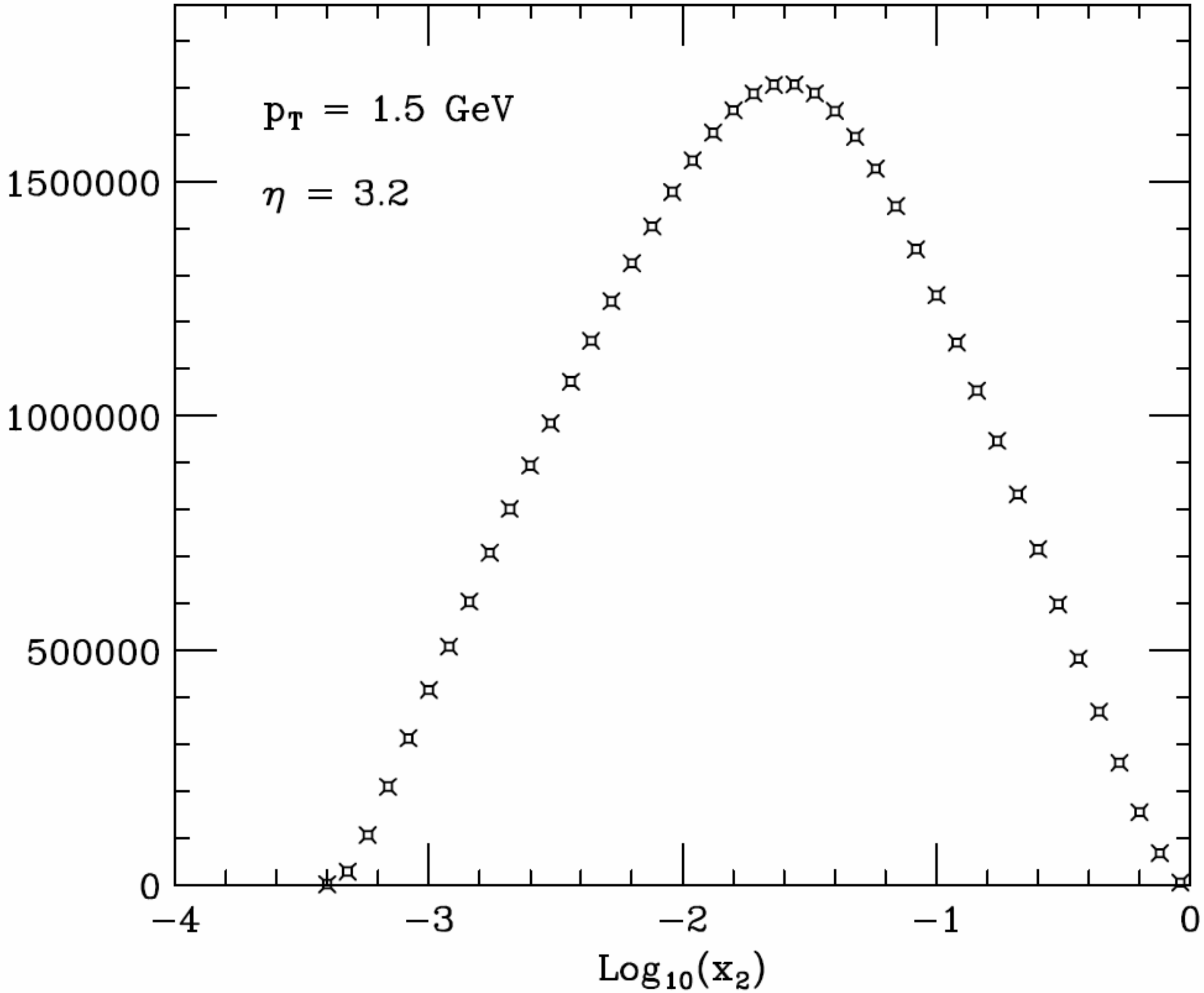
HERMES, COMPASS

• can also use high-p_T hadrons

SMC, HERMES, COMPASS

Bravar, Kotzinian, v. Harrach

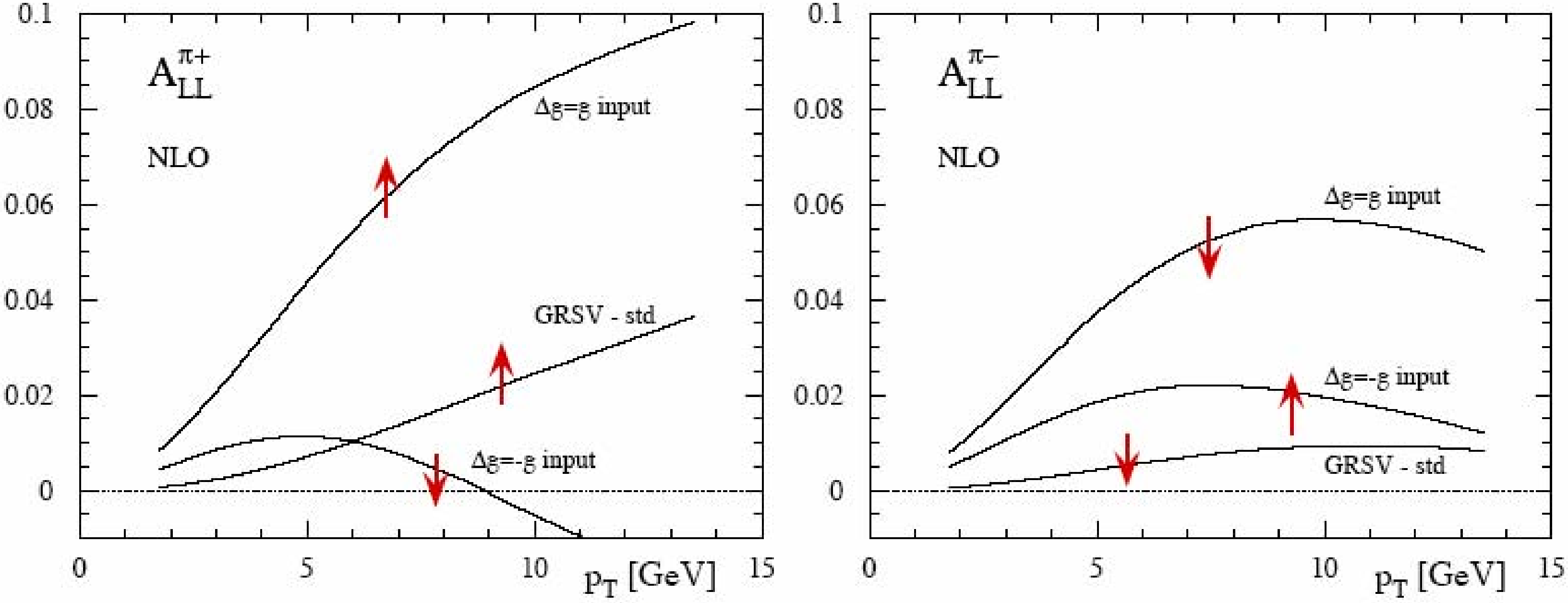
(from Guzey, Strikman ,WV)



π^+ and π^-

positive Δg: $A_{LL}^{\pi^+} > A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^+} < A_{LL}^{\pi^0}$

positive Δg: $A_{LL}^{\pi^-} < A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^-} > A_{LL}^{\pi^0}$



…only at $p_T > 5$ GeV, good statistics required